

How to draw a rough sketch of the parametric curve $x = f(t)$ **quickly while plotting only a few points**
 $y = g(t)$

using $x = 3 + 2t - t^2$, $t \in (-\infty, \infty)$ **as an example**
 $y = t^2 + 2t - 8$

- [1] Sketch the graphs $x = f(t)$ and $y = g(t)$ on two separate sets of axes.

(On both graphs, the horizontal axis is t .)

$$x = 3 + 2t - t^2 = (3 - t)(1 + t)$$

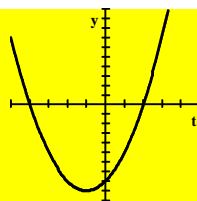
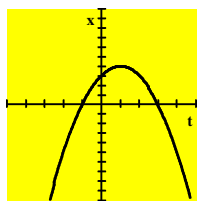
upside down parabola

t -intercepts -1 and 3

$$y = t^2 + 2t - 8 = (t - 2)(t + 4)$$

upright parabola

t -intercepts -4 and 2



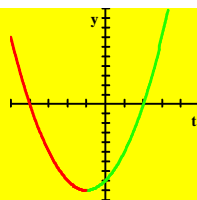
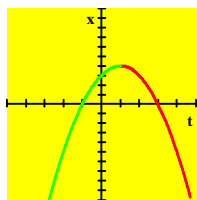
- [2] Find the t -values at which either graph changes general direction (changes from either increasing, constant or decreasing to another general direction) or makes a sudden discontinuous jump.

$$x = 3 + 2t - t^2$$

changes from increasing
to decreasing at $t = 1$

$$y = t^2 + 2t - 8$$

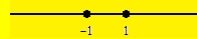
changes from decreasing
to increasing at $t = -1$



- [3] On a number line for the domain of the parametric equations, mark down the values of t found in step [2].

domain of the parametric equations: $t \in (-\infty, \infty)$

number line:



- [4] For each interval that the number line is subdivided into in step [3], determine whether x and y are increasing or decreasing from the graphs in step [2].

Determine the direction the curve is oriented by noting that

if x is increasing, the curve is going to the right,

if x is decreasing, the curve is going to the left,

if y is increasing, the curve is going upwards, and

if y is decreasing, the curve is going downwards.

$t < -1$: x is increasing, y is decreasing, the curve is going to the right and downwards

$-1 < t < 1$: x is increasing, y is increasing, the curve is going to the right and upwards

$t > 1$: x is decreasing, y is increasing, the curve is going to the left and upwards

- [5] At the values of t found in step [2], find the exact co-ordinates of the graph using the parametric equations. At the endpoints of the domain, find general approximations of the co-ordinates of the graph (eg. using “ $\rightarrow -\infty$ ”, “ $\rightarrow \infty$ ”, “just above 0”, “just below 0” etc.).

Note that

$x = 0$ means “on the y – axis” (ie. y – intercept)

x just above 0 means “just to the right of the y – axis”

x just below 0 means “just to the left of the y – axis”

$y = 0$ means “on the x – axis” (ie. x – intercept)

y just above 0 means “just above the x – axis”

y just below 0 means “just below the x – axis”

$x \rightarrow -\infty$ means “the left side of the grid”

$x \rightarrow \infty$ means “the right side of the grid”

$y \rightarrow -\infty$ means “the bottom of the grid”

$y \rightarrow \infty$ means “the top of the grid”

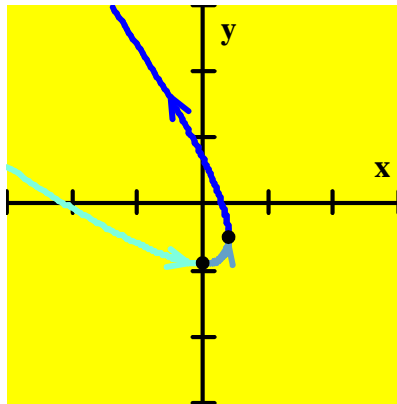
$t \rightarrow -\infty$: $(x, y) \rightarrow (-\infty, \infty)$, the curve is starting from the top left corner of the grid

$t = -1$: $(x, y) = (0, -9)$

$t = 1$: $(x, y) = (4, -5)$

$t \rightarrow \infty$: $(x, y) \rightarrow (-\infty, \infty)$, the curve is going off the top left corner of the grid

- [6] Combine the points in step [5] with the directions in step [4] to draw a rough sketch of the curve.



NOTE: When both x and y are going towards ∞ or $-\infty$,

it is useful to notice which variable has the larger absolute value.

If x has the larger absolute value, the graph is closer to the x – axis than the y – axis.

If y – has the larger absolute value, the graph is closer to the y – axis than the x – axis.